**The Time Value of Money**

A dollar on hand today is worth more than a dollar to be received in the future because the dollar on hand today can be invested to earn interest to yield more than a dollar in the future. The Time Value of Money mathematics quantify the value of a dollar through time. This, of course, depends upon the rate of return or interest rate which can be earned on the investment.

The Time Value of Money has applications in many areas of Corporate Finance including Capital Budgeting, Bond Valuation, and Stock Valuation. For example, a bond typically pays interest periodically until maturity at which time the face value of the bond is also repaid. The value of the bond today, thus, depends upon what these future cash flows are worth in today's dollars.

The Time Value of Money concepts will be grouped into two areas: Future Value and Present Value. Future Value describes the process of finding what an investment today will grow to in the future. Present Value describes the process of determining what a cash flow to be received in the future is worth in today's dollars.

**Future Value**

The Future Value of a cash flow represents the amount, at some time in the future, that an investment made today will grow to if it is invested to earn a specific interest rate. For example, if you were to deposit $100 today in a bank account to earn an interest rate of 10% compounded annually, this investment will grow to $110 in one year. This can be shown as follows:

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| **Year 1**  |
| $100(1 + 0.10) = $110 |

At the end of two years, the initial investment will have grown to $121. Notice that the investment earned $11 in interest during the second year, whereas, it only earned $10 in interest during the first year. Thus, in the second year, interest was earned not only on the initial investment of $100 but also on the $10 in interest that was paid at the end of the first year. This occurs because the interest rate in the example is a compound interest rate.

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| Compound Interest  |
| Under compound interest, interest is earned not only on the initial principal but also on the accumulated interest. Interest begins to be earned on the accumulated interest as soon as it is paid, which occurs at the end of each compounding period. This is in contrast to simple interest, under which interest is only earned on the initial principal. Valuations should generally be based on compound interest because, after the interest has been paid, the full amount, i.e., the initial principal plus interest, could be withdrawn and reinvested elsewhere. Thus, interest on the new investment would be earned on the full amount.  |

The interest rate in the example is 10% compounded annually. This implies that interest is paid annually. Thus the balance in the account was $110 at the end of the first year. Thus, in the second year the account pays 10% on the initial principal of $100 and the $10 of interest earned in the first year. Thus, the $121 balance in the account after two years can be computed as follows:

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| **Year 2**  |
| $110(1+0.10) = $121 or$100(1+0.10)(1+0.10) = $121 or$100(1+0.10)2 = $121 |

If the money was left in the account for one more year, interest would be earned on $121, i.e., the initial principal of $100, the $10 in interest paid at the end of year 1, and the $11 in interest paid at the end of year 2. Thus the balance in the account at the end of year three is $133.10. This can be computed as follows:

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| **Year 3**  |
| $121(1+0.10) = $133.10 or$100(1+0.10) (1+0.10) (1+0.10) = $133.10 or$100 (1+0.10)3 = $133.10 |

A pattern should be becoming apparent. The Future Value of an initial investment at a given interest rate compounded annually at any point in the future can be found using the following equation:



where

* FVt = the Future Value at the end of year t,
* CF0 = the initial investment,
* r = the annually compounded interest rate, and
* t = the number of years.

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| **Future Value Example**  |
| Find the Future Value at the end of 4 years of $100 invested today at an interest rate 10%.**Solution:**http://www.prenhall.com/divisions/bp/app/cfl/TVM/images/FutureValueA.gif |

**Present Value**

Present Value describes the process of determining what a cash flow to be received in the future is worth in today's dollars. Therefore, the Present Value of a future cash flow represents the amount of money today which, if invested at a particular interest rate, will grow to the amount of the future cash flow at that time in the future. The process of finding present values is called *Discounting* and the interest rate used to calculate present values is called the *discount rate*. For example, the Present Value of $100 to be received one year from now is $90.91 if the discount rate is 10% compounded annually. This can be demonstrated as follows: *(Refer to the* [*Future Value*](http://www.prenhall.com/divisions/bp/app/cfl/TVM/FutureValue.html) *page if you are unfamiliar with the calculations.)*

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| **One Year**  |
| $90.91(1 + 0.10) = $100 or$90.91 = $100/(1 + 0.10) |

Notice that the Future Value Equation is used to describe the relationship between the present value and the future value. Thus, the Present Value of $100 to be received in two years can be shown to be $82.64 if the discount rate is 10%.

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| **Two Years**  |
| $82.64(1 + 0.10)2 = $100 or$82.64 = $100/(1 + 0.10)2 |

A pattern should be becoming apparent. The following equation can be used to calculate the Present Value of a future cash flow given the discount rate and number of years in the future that the cash flow occurs. *(This equation can be obtained algebraically from the Future Value Equation.)*



where

* PV = Present Value
* CFt = Future Cash Flow which occurs t years from now
* r = the interest or discount rate
* t = the number of years

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| **Present Value Example**  |
| Find the Present Value of $100 to be received 3 years from today if the interest rate is 10%.**Solution:**http://www.prenhall.com/divisions/bp/app/cfl/TVM/images/PresentValueA.GIF |
|  | http://www.prenhall.com/divisions/bp/app/cfl/images/space.gifCash Flow StreamsPresent ValueThe Present Value of a Cash Flow Stream is equal to the sum of the Present Values of the individual cash flows. To see this, consider an investment which promises to pay $100 one year from now and $200 two years from now. If an investor were given a choice of this investment or two alternative investments, one promising to pay $100 one year from now and the other promising to pay $200 two years from now, clearly, he would be indifferent between the two choices. *(Assuming that the investments were all of equal risk, i.e., the discount rate is the same.)* This is because the cash flows that the investor would receive at each point in time in the future are the same under either alternative. Thus, if the discount rate is 10%, the Present Value of the investment can be found as follows:

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| **Present Value of the Investment**  |
| PV = $100/(1 + 0.10) + $200/(1 + 0.10)2 PV = $90.91 + $165.29 = $256.20  |

The following equation can be used to find the Present Value of a Cash Flow Stream. http://www.prenhall.com/divisions/bp/app/cfl/TVM/images/PVCFEqNew.gifwhere * PV = the Present Value of the Cash Flow Stream,
* CFt = the cash flow which occurs at the end of year t,
* r = the discount rate,
* t = the year, which ranges from zero to n, and
* n = the last year in which a cash flow occurs.

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| **Present Value Example**  |
| Find the Present Value of the following cash flow stream given that the interest rate is 10%.http://www.prenhall.com/divisions/bp/app/cfl/TVM/images/PVCFA.gif**Solution:**http://www.prenhall.com/divisions/bp/app/cfl/TVM/images/PVCFASol.gifTop of FormBottom of Form |

Top of FormBottom of FormFuture ValueThe Future Value of a Cash Flow Stream is equal to the sum of the Future Values of the individual cash flows. For example, consider an investment which promises to pay $100 one year from now and $200 two years from now. Given that the discount rate is 10%, the Future Value at the end of year 2 of the investment can be found as follows:

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| **Future Value of the Investment**  |
| FV2 = $100(1 + 0.10) + $200 FV2 = $110.00 + $200.00 = $310.00  |

As of year 2, the $100 received at the end of year 1 would have earned interest for one year while the $200 received at the end of year 2 would not yet have earned any interest. Thus, the Future Value at the end of year 2, *i.e.,* immediately after the $200 cash flow was received, is $310.00. The following equation can be used to find the Future Value of a Cash Flow Stream at the end of year t. http://www.prenhall.com/divisions/bp/app/cfl/TVM/images/FVCFEqNew.gifwhere * FVt = the Future Value of the Cash Flow Stream at the end of year t,
* CFt = the cash flow which occurs at the end of year t,
* r = the discount rate,
* t = the year, which ranges from zero to n, and
* n = the last year in which a cash flow occurs.

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| **Future Value Example**  |
| Find the Future Value at the end of year 4 of the following cash flow stream given that the interest rate is 10%.http://www.prenhall.com/divisions/bp/app/cfl/TVM/images/PVCFA.gif**Solution:**http://www.prenhall.com/divisions/bp/app/cfl/TVM/images/FVCFASol.gif |

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# Annuities

An Annuity is a cash flow stream which adheres to a specific pattern. Namely, an Annuity is a cash flow stream in which the cash flows are level (*i.e.,* all of the cash flows are equal) and the cash flows occur at a regular interval. The annuity cash flows are called *annuity payments* or simply *payments*. Thus, the following cash flow stream is an annuity.

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| **Figure 1**  |
| http://www.prenhall.com/divisions/bp/app/cfl/images/Annuity.gif |

While, the following cash flow stream is not an annuity because the payments do not occur at a regular interval.

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| **Figure 2**  |
| http://www.prenhall.com/divisions/bp/app/cfl/images/NotAnnuity.gif |

When a cash flow stream is of the form given in **Figure 1**, *i.e.,* an annuity, the process of finding the [Present Value](http://www.prenhall.com/divisions/bp/app/cfl/TVM/PresentValue.html) or [Future Value](http://www.prenhall.com/divisions/bp/app/cfl/TVM/FutureValue.html) of the cash flow stream is greatly simplified.

## Present Value of an Annuity

The Present Value of an Annuity is equal to the sum of the present values of the annuity payments. This can be found in one step through the use of the following equation:



where

* PVA = The Present Value of the Annuity
* PMT = The Annuity Payment
* r = The Interest or Discount Rate
* t = The Number of Years (also the Number of Annuity Payments)

Consider the annuity of $100 per year for five years given in **Figure 1**. If the discount rate is equal to 10%, then the Present Value of the Annuity can be found as follows:

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| **Present Value of the Annuity**  |
| http://www.prenhall.com/divisions/bp/app/cfl/TVM/images/PVAEx1.gif |

## Future Value of an Annuity

The Future Value of an Annuity is calculated at the end of the period in which the last annuity payment occurs. The Future Value of the Annuity is equal to the sum of the future values of the individual annuity payments at that time. Thus, the future value of a five year annuity is computed at the end of year five. This can be found in one step through the use of the following equation:



where

* FVA = The Present Value of the Annuity
* PMT = The Annuity Payment
* r = The Interest or Discount Rate
* t = The Number of Years (also the Number of Annuity Payments)

Consider the annuity of $100 per year for five years given in **Figure 1**. If the discount rate is equal to 10%, then the Future Value of this Annuity at the end of period five can be found as follows:

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| **Future Value of the Annuity**  |
| http://www.prenhall.com/divisions/bp/app/cfl/TVM/images/FVAEx1.gif |

# Other Compounding Periods

In the real world, interest rates are often compounded more often than once per year. By convention, interest rates are quoted on an annual basis. An interest rate, quoted on an annual basis, which is compounded more often than once per year is called a *nominal rate, stated rate, quoted rate,* or *annual percentage rate (APR).* For example, mortgages typically require monthly payments and, therefore, the interest rates quoted on mortgages are compounded monthly. Thus, the nominal interest rate on a mortgage might be 12% compounded monthly. However, the relevant rate for valuations is the periodic rate. The periodic rate is computed by dividing the nominal rate by the number of compounding periods per year.



where

* r = the rate per period,
* rnom = the nominal rate, and
* m = the number of compounding periods per year.

Thus a 12% nominal rate compounded monthly is equivalent to a periodic rate of 1% per month.

The following sections of this page demonstrate how to convert a nominal rate into an equivalent rate that is compounded annually and provide versions of the Present Value and Future Value formulas for use with interest rates compounded more often than once per year. The page concludes with a discussion of continuous compounding.

## EAR - Effective or Equivalent Annual Rate

The Effective or Equivalent Annual Rate (EAR) is the interest rate compounded annually that is equivalent to a nominal rate compounded more than once per year. In other words, present and future values computed using the EAR will be the same as those computed using the nominal rate. The EAR is computed as follows:



* EAR = the Equivalent or Effective Annual Rate,
* rnom = the nominal interest rate,
* m = the number of compounding periods per year, and

Moreover, it is not proper to directly compare interest rates which have a particular compounding frequency with those that have a different compounding frequency, *e.g.,*, comparing 10.1% compounded semiannually with 10% compounded quarterly. This problem can be overcome by finding the EAR for each of the rates and then comparing the EARs.

First, let's find the EAR for 10.1% compounded semiannually. Here, m equals 2.

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| **EAR for 10.1% compounded semiannually**  |
| http://www.prenhall.com/divisions/bp/app/cfl/TVM/images/EAREx1.gif |

Now, let's find the EAR for 10% compounded quarterly. Here m = 4.

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| **EAR for 10% compounded quarterly**  |
| http://www.prenhall.com/divisions/bp/app/cfl/TVM/images/EAREx2.gif |

Thus, we see that 10% compounded quarterly is actually a higher interest rate than 10.1% compounded semiannually. Given a choice, we would prefer to invest at 10% compounded quarterly.

## Present Value

The Present Value of a future cash flow when the interest rate is compounded m times per year can be calculated as follows:



where

* PV = the Present Value,
* CFt = the cash flow which occurs at the end of year t,
* rnom = the nominal interest rate,
* m = the number of compounding periods per year, and
* t = the number of years.
* Thus, mt = the number of compounding periods in t years.

In the earlier discussion of [Present Value](http://www.prenhall.com/divisions/bp/app/cfl/TVM/PresentValue.html) the interest rate was compounded annually and there was one compounding period per year. In that case m = 1. Thus, our earlier Present Value formula is actually just a special case of this formula since under annual compounding the rate per period is the same as the nominal rate.

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| **Present Value Example**  |
| Find the Present Value of $100 to be received 3 years from today if the interest rate is 12% compounded quarterly.**Solution:**http://www.prenhall.com/divisions/bp/app/cfl/TVM/images/PVmA.gif |

## Future Value

The Future Value of a future cash flow when the interest rate is compounded m times per year can be calculated as follows:



where

* FV = the Future Value,
* CF0 = the cash flow which occurs at time 0,
* rnom = the nominal interest rate,
* m = the number of compounding periods per year, and
* t = the number of years.
* Thus, mt = the number of compounding periods in t years.

Thus, the earlier Future Value formula is actually just a special case of this formula since under annual compounding (*i.e.,* when m = 1) the rate per period is the same as the nominal rate.

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| **Future Value Example**  |
| Find the Future Value of 3 years from now of $100 invested today at an interest rate of 10% compounded semiannually.**Solution:**http://www.prenhall.com/divisions/bp/app/cfl/TVM/images/FVmA.gif |

## Present Value of an Annuity

The Present Value of an Annuity when the payments occur m times per year and the interest rate is compounded m times per year can be calculated as follows:



where

* PVA = the Present Value,
* PMT = the Annuity Payment which occurs m times per year,
* rnom = the nominal interest rate,
* m = the number of compounding periods per year, and
* t = the number of years.
* Thus, mt = the number of payments and compounding periods in t years.

This formula can only be applied when the frequency of the annuity payments is the same as the compounding period for the interest rate. For example, if the annuity has quarterly payments the interest rate must be compounded quarterly (m = 4).

Thus, the earlier Present Value on an Annuity formula is actually just a special case of this formula since under annual compounding (*i.e.,* when m = 1) the rate per period is the same as the nominal rate.

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| **Present Value of an Annuity Example**  |
| Find the Present Value of an annuity of $100 per month for 2 years if the interest rate is 12% compounded monthly.**Solution:**http://www.prenhall.com/divisions/bp/app/cfl/TVM/images/PVAmA.gif |

## Future Value of an Annuity

The Future Value of an Annuity when the payments occur m times per year and the interest rate is compounded m times per year can be calculated as follows:



where

* FVAt = the Future Value of the annuity at the end of year t,
* PMT = the Annuity Payment which occurs m times per year,
* rnom = the nominal interest rate,
* m = the number of compounding periods per year, and
* t = the number of years.
* Thus, mt = the number of payments and compounding periods in t years.

This formula can only be applied when the frequency of the annuity payments is the same as the compounding period for the interest rate. For example, if the annuity has quarterly payments the interest rate must be compounded quarterly (m = 4). As with the earlier formula, the Future Value is computed at the end of the period in which the last annuity payment occurs.

Thus, the earlier Future Value on an Annuity formula is actually just a special case of this formula since under annual compounding (*i.e.,* when m = 1) the rate per period is the same as the nominal rate.

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| **Future Value of an Annuity Example**  |
| Find the Future Value at the end of 3 years of an annuity of $100 per quarter for 3 years if the interest rate is 8% compounded quarterly.**Solution:**http://www.prenhall.com/divisions/bp/app/cfl/TVM/images/FVAmA.gif |

**Time Value of Money Equations**

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| Annual Compounding |
| Present Value: | http://www.prenhall.com/divisions/bp/app/cfl/TVM/images/PVEqNew.gif |
| Future Value: | http://www.prenhall.com/divisions/bp/app/cfl/TVM/images/FVEqNew.gif |
| Present Value of a Cash Flow Stream: | http://www.prenhall.com/divisions/bp/app/cfl/TVM/images/PVCFEqNew.gif |
| Future Value of a Cash Flow Stream: | http://www.prenhall.com/divisions/bp/app/cfl/TVM/images/FVCFEqNew.gif |
| Present Value of an Annuity: | http://www.prenhall.com/divisions/bp/app/cfl/TVM/images/PVAEqNew.gif |
| Future Value of an Annuity: | http://www.prenhall.com/divisions/bp/app/cfl/TVM/images/FVAEqNew.gif |
| Other Compounding Periods |
| Rate per Period: | http://www.prenhall.com/divisions/bp/app/cfl/TVM/images/RatePPEqNew.gif |
| Effective Annual Rate: | http://www.prenhall.com/divisions/bp/app/cfl/TVM/images/EAREqNew.gif |
| Present Value: | http://www.prenhall.com/divisions/bp/app/cfl/TVM/images/PVmEqNew.gif |
| Future Value: | http://www.prenhall.com/divisions/bp/app/cfl/TVM/images/FVmEqNew.gif |
| Present Value of an Annuity: | http://www.prenhall.com/divisions/bp/app/cfl/TVM/images/PVAmEqNew.gif |
| Future Value of an Annuity: | http://www.prenhall.com/divisions/bp/app/cfl/TVM/images/FVAmEqNew.gif |

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