Activity – Introduction to Mass Defect and Nuclear Binding Energy

As our AP Physics II course dives further into the physical laws governing the motion and behavior of subatomic particles, our curriculum begins to cross paths with AP Chemistry topics.

This video does a great job of outlining the foundations governing fission, fusion, and the physics of mass-defect style problems.

Crash Course – Nuclear Physics
https://www.youtube.com/watch?v=lUhJL7o6_cA&list=PL8dPuuaLjXtN0ge7yDk_UA0ldZJdhwkoV&index=46&t=499s

Helpful Constants When Tackling Mass Defect-Style Problems Include:

$Mass_{Proton}$ = 1.00728 amu — 1 amu = 1.6606 X 10^{-27} kg

$Mass_{Neutron}$ = 1.00867 amu — 931 MeV per 1.00 amu

Questions

(1) What are the four fundamental forces in physics?

(2) Provide an example of the graph depicting binding energy per nucleon, (Y-axis), and number of nucleons, (X-axis). (This can be easily researched and depicted). Note, especially, specific elements at which the slope experiences a significant alteration).

(3) When, (on the X-Axis), does the sketch begin trending downward? Why does this trend occur at this location on the X-Axis?

(4) What force is responsible for holding the nucleon components of an atom's nucleus together?

Listed below are some common particles which are involved in nuclear reactions, (fission or fusion).

Name	Symbol(s)	Representation	Description
Alpha particle	${}^{4}_{2}He$ or ${}^{4}_{2}\alpha$		(High-energy) helium nuclei consisting of two protons and two neutrons
Beta particle	${}^{0}_{1}e$ or ${}^{0}_{-1}\beta$		(High-energy) electrons
Positron	${}^{0}_{+1}e$ or ${}^{0}_{+1}\beta$		Particles with the same mass as an electron but with 1 unit of positive charge
Proton	${}^{1}_{1}H$ or ${}^{1}_{1}p$		Nuclei of hydrogen atoms
Neutron	${}^{1}_{0}n$		Particles with a mass approximately equal to that of a proton but with no charge
Gamma ray	γ	 γ	Very high-energy electromagnetic radiation

On the next two pages is content regarding your approach to attacking mass defect and nuclear binding energy-style questions.

The energy required to break down a nucleus into its component nucleons is called the *nuclear binding energy*.

^{63}Cu + Energy → 29 p^+ + 34 n^o

Nuclear binding energies are usually expressed in terms of kJ/mole of nuclei or MeV's/nucleon. Calculation of the nuclear binding energy involves the following three steps:

(1) Determining the Mass Defect Between Reactants and Products
(2) Conversion of Mass Defect into Energy ($e = mc^2$; remember, we should use kilograms for mass rather than amu's when plugging into Einstein's famous equation)
(3) Expressing Nuclear Binding Energy as Energy per Mole or Energy per Nucleon

Determining the Mass Defect

The difference between the mass of a nucleus and the sum of the masses of the nucleons of which it is composed is called the *mass defect.* Three things need to be known in order to calculate the mass defect:

•The actual mass of the nucleus
•The composition of the nucleus (number of protons and of neutrons)
•The masses of a proton and of a neutron (which we know from the table provided on the previous page)

To calculate the mass defect:
•Add up the masses of each proton and of each neutron that make up the nucleus
•Subtract the actual mass of the nucleus from the combined mass of the components to obtain the mass defect

Example: Find the mass defect of a copper-63 nucleus if the actual mass of a copper-63 nucleus is 62.91367 amu.

First, find the composition of the copper-63 nucleus and determine the combined mass of its components.
Copper has 29 protons and copper-63 also has, (63 - 29), 34 neutrons.

The mass of a proton is 1.00728 amu and a neutron is 1.00867 amu.

The combined mass is calculated:

29 protons(1.00728 amu/proton) + 34 neutrons(1.00867 amu/neutron)

or 63.50590 amu

Calculate the mass defect.

Dm = 63.50590 amu - 62.91367 amu = 0.59223 amu

Conversion of Mass Defect into Energy

To convert the mass defect into energy:

•Convert the mass defect into kilograms (1 amu = 1.6606×10^{-27} kg)
•Convert the mass defect into its energy equivalent using Einstein's equation.

Example: Determine the binding energy of the copper-63 atom.

•Convert the mass defect (calculated in the previous example) into kg.

(0.59223 amu/nucleus)(1.6606×10^{-27} kg/amu) = 9.8346×10^{-28} kg (per nucleus)

Convert this mass into energy using $E = mc^2$, where c = 2.9979×10^8 m/s.

E = (9.8346×10^{-28} kg/nucleus)(2.9979×10^8 m/s)^2 = 8.8387×10^{-11} J (per nucleus)

Expressing Nuclear Binding Energy as Energy per Mole of Atoms, or as Energy per Nucleon

(931 MeV per 1.00 amu)

The energy calculated in the previous example is the nuclear binding energy. However, nuclear binding energy is often expressed as kJ/mol of nuclei or as MeV/nucleon.

To convert the energy to kJ/mol of nuclei we will simply employ the conversion factors for converting joules into kilojoules (1 kJ = 1000 J) and for converting individual particles into moles of particles (Avogadro's Number).

$(8.8387 \times 10^{-11}$ J/nucleus)(1 kJ/1000 J)$(6.022 \times 10^{23}$ nuclei/mol) = 5.3227×10^{10} kJ/mol of nuclei

To convert the binding energy to MeV (megaelectron volts) per nucleon we will employ the conversion factor for converting joules into MeV (1 MeV = 1.602×10^{-13} J) and the number of nucleons (protons and neutrons) which make up the nucleus.

$(8.8387 \times 10^{-11}$ J/nucleus)[1 MeV/$(1.602 \times 10^{-13}$ J)](1 nucleus/63 nucleons) = 8.758 MeV/nucleon

Activity – Mass Defect & Nuclear Binding Energy-Style Questions

Hopefully, you've referenced the previous three pages which outlines the methods by which we attack Nuclear Binding Energy & Mass Defect style problems. They characterize a section of our nuclear physics curriculum, especially laying the foundation for energy released during fission or fusion reactions, (which we'll get to in a moment).

You will require a periodic table which approximates atomic mass pretty aggressively, (as in, to four decimal places past the decimal point).

Helpful Constants:
•Mass of a proton: 1.007825 units
•Mass of a neutron: 1.008665 units
•1 amu = 931 MeV

1. Tritium is an isotope of hydrogen. The mass of the tritium isotope, H-3, is 3.0160490 amu.

a. What is the mass defect of this isotope?

b. What is the binding energy of this isotope?

c. Find the binding energy per nucleon.

2. The mass of a C-12 nucleus is 12.00000 units.

a. What is the mass defect of this nucleus?

b. What is the binding energy of this nucleus?

c. Find the binding energy per nucleon.

3. An oxygen isotope, O-16 , has a mass of 15.99491 units.

a. What is the mass defect of this isotope

b. What is the binding energy of this isotope?

c. Find the binding energy per nucleon.

4. The mass of an iron-56 nucleus is 55.92066 units.

a. What is the mass defect of this nucleus?

b. What is the binding energy of the nucleus?

c. Find the binding energy per nucleon.

5. The binding energy of Helium-4 is 28 MeV.

a. What is the mass of a helium nucleus (round to 5 decimal places).

Answers:

Answer Key: 1.0.009106amu, 8.48MeV, 2.83MeV 2. 0.98940amu, 92.1MeV, 7.68MeV 3. 0.13701amu, 128MeV, 8MeV 4. 0.54274amu, 505MeV, 9.017MeV 5. 4.00290amu

Activities – Nuclear Reactions, (Fission and Fusion Reactions)

Nuclear physics topics involving fission and fusion events share a lot with our previous investigations into binding energy and mass defect. First a couple of crash course videos, (from the chemistry section of their video bank), will inform the foundations for our investigations into fission and fusion.

Crash Course # 38 - 39
https://www.youtube.com/watch?v=KWAsz59F8gA&list=PL8dPuuaLjXtPHzzYuWy6fYEaX9mQQ8oGr&t=0s

https://www.youtube.com/watch?v=FU6y1XIADdg&list=PL8dPuuaLjXtPHzzYuWy6fYEaX9mQQ8oGr&index=40

Helpful constants to know --
$E = (m)(c)^2$
9.31 X 10^8 eV in 1.0 amu
Nuclear Mass = Atomic Mass – Mass Electrons

Two Important Features of a Nuclear Reaction:
•Mass number is conserved
•Charge is conserved

$^{2}_{1}H$ Deuteron

Reaction can be spontaneous if discrete levels of mass are gained in the reaction.
Reaction is not spontaneous if discrete levels of mass are lost in the reaction.

To find binding energy, $Mass_{Atom}$ – $Mass_{Added\ Parts}$ = $Mass_{Differential}$
($Mass_{Defect}$)(9.31 X 10^8 eV) = Binding Energy (in eV)

Common Particles Involved in Nuclear Reactions:

Type of Radiation	Alpha particle	Beta particle	Gamma ray
Symbol	α or $^{4}_{2}\alpha$ or $^{4}_{2}He$	β or β^-	γ (can look different, depends on the font)
Mass *(atomic mass units)*	4	1/2000	0
Charge	+2	-1	0
Speed	slow	fast	very fast (speed of light)

(1) A mercury-198 nucleus is bombarded by a neutron, which causes a nuclear reaction:

$$^{1}_{0}n + ^{198}_{80}Hg \Rightarrow ^{197}_{79}Au + \_\_\_\_\_$$

What is the unknown product particle?

Use the below information to answer Questions 2 – 3.
In the first nuclear reaction using a particle accelerator, accelerated protons bombarded lithium atoms, producing alpha particles and energy. The energy resulted from a conversion of mass into energy. The reaction is written as follows:

$$ {}^{1}_{1}\mathrm{H} + {}^{7}_{3}\mathrm{Li} \Rightarrow 2\,{}^{4}_{2}\mathrm{He} + \text{Energy} $$

Data Table

Particle	Symbol	Mass (u)
Proton	H	1.00783
Lithium Atom	Li	7.01600
Alpha Particle	He	4.00260

(2) Determine the difference between the total mass of a proton plus a lithium atom, and the total mass of two alpha particles, in universal mass units.

(3) Determine the energy produced in the reaction of a proton with a lithium atom.

Use the below information to answer Questions 4 – 5.
The process that powers the Sun is the below fusion reaction:

$$ 4\,{}^{1}_{1}\mathrm{H} \Rightarrow {}^{4}_{2}a + 2\,e^{+} + 2v_{e} + y $$

- •Mass Proton = 1.00728 amu
- •Mass Alpha Particle = 4.0015 amu
- •Mass Positron = 0.0005486 amu

The mass of an electron neutrino, (v_e), is negligible.

(4) Show that this reaction releases energy.

(5) How much energy is released?

(6) Can the following nuclear reaction occur spontaneously?

$${}^{4}_{2}a + {}^{14}_{7}\mathrm{N} \Rightarrow {}^{17}_{8}\mathrm{O} + {}^{1}_{1}\mathrm{H}$$

•Mass of an Alpha Particle = 4.0015 amu
•Mass of a Nitrogen Nucleus = 13.9992 amu
•Mass of an Oxygen Nucleus = 16.9947 amu
•Mass of a Proton = 1.00728 amu

(7) Given the following masses, what is the mass defect and binding energy for the helium atom? (Electrons are not factored).

Particle	Mass
Proton	1.0073 amu
Neutron	1.0087 amu
Helium Atom	4.0016 amu

(8) What is the maximum wavelength of electromagnetic radiation that could be used to photo-disintegrate a deuteron?

Proton amu = 1.00728 amu
Neutron amu = 1.00867 amu
Deuteron amu = 2.01356 amu

(9) The atomic mass of aluminum-27 is 26.9815 amu. What is its nuclear binding energy **per nucleon**? (Mass of electron = 0.0005486 amu).

Answers:

(1) $^{2}_{1}H$ Deuteron

(2) 0.01863 amu (3) 1.734 X 10^{7} eV

(4) Mass Reactants: 4.02912 amu
Mass Products: 4.002597 amu
Mass Defect = .026523 amu is lost; energy is released
(5) 24693000 eV
(6) Mass Reactants = 18.0007 amu
Mass Products = 18.00198 amu
-0.00128 amu lost; non-spontaneous occurance
(7) Mass Defect = 0.0304 amu; binding energy = 2.83 X 10^{7} eV
(8) Mass Defect = 0.00239 amu
Binding Energy = 2225000 eV
Use E = (hc)/λ to solve for λ; 5.56 X 10^{-13} m
(9) Mass Defect = 0.24163 amu; convert mass to energy = 224960000 ev; this is energy per atom; per nucleon is divided by 27; 8.3 X 10^{6} eV