Bresenham’s Line Drawing Algorithm

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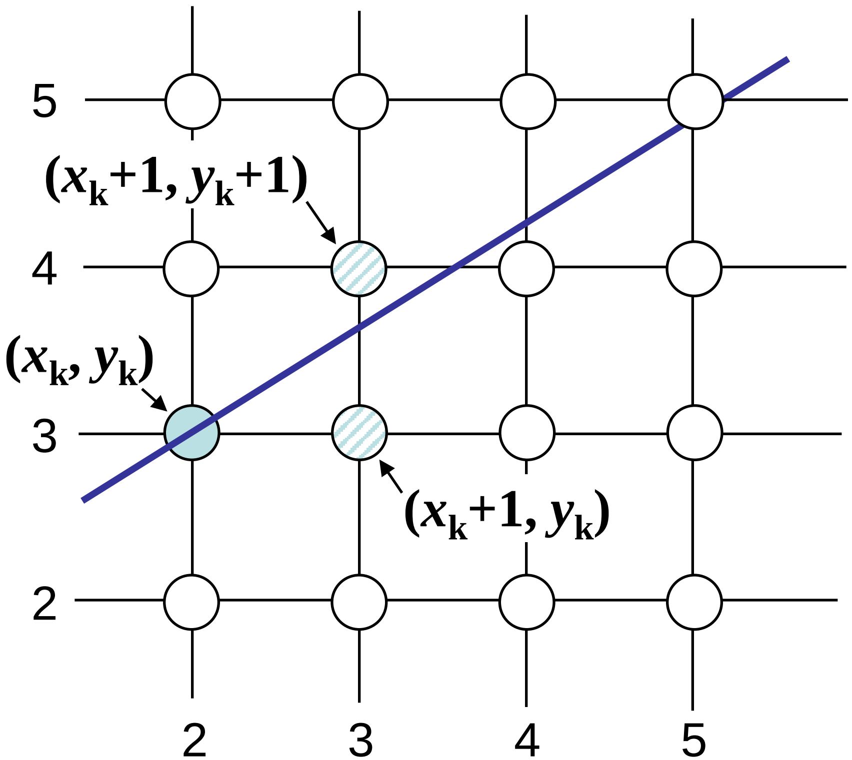
The Bresenham's Line Algorithm

- The Bresenham's algorithm is another incremental scan conversion algorithm
- The big advantage of this algorithm is that it uses only integer calculations

Jack Bresenham worked for 27 years at IBM before entering academia. Bresenham developed his famous algorithms at IBM in the early 1960s

The Big Idea

Move across the *x* axis in unit intervals and at each step choose between two different *y*-coordinates

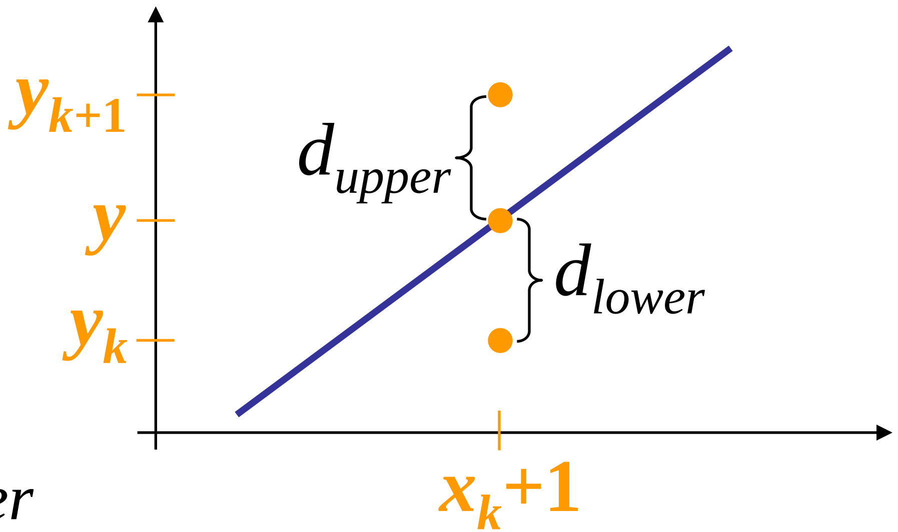


For example, from position (2, 3) we have to choose between (3, 3) and (3, 4)

We would like the point that is closer to the original line

Deriving The Bresenham Line Algorithm

At sample position x_k+1 the vertical separations from the mathematical line are labelled d_{upper} and d_{lower}



The y coordinate on the mathematical line at x_k+1 is:

$$y = m(x_k + 1) + b$$

Deriving The Bresenham Line Algorithm (cont…)

So, d_{upper} and d_{lower} are given as follows:

$$d_{lower} = y - y_k$$
$$= m(x_k + 1) + b - y_k$$

and:

$$d_{upper} = (y_k + 1) - y$$
$$= y_k + 1 - m(x_k + 1) - b$$

We can use these to make a simple decision about which pixel is closer to the mathematical line

Deriving The Bresenham Line Algorithm (cont…)

This simple decision is based on the difference between the two pixel positions:

$$d_{lower} - d_{upper} = 2m(x_k + 1) - 2y_k + 2b - 1$$

Let's substitute m with $\Delta y / \Delta x$ where Δx and Δy are the differences between the end-points:

$$\Delta x(d_{lower} - d_{upper}) = \Delta x(2\frac{\Delta y}{\Delta x}(x_k + 1) - 2y_k + 2b - 1)$$

$$= 2\Delta y \cdot x_k - 2\Delta x \cdot y_k + 2\Delta y + \Delta x(2b - 1)$$

$$= 2\Delta y \cdot x_k - 2\Delta x \cdot y_k + c$$

Deriving The Bresenham Line Algorithm (cont…)

So, a decision parameter p_k for the kth step along a line is given by:

$$p_k = \Delta x(d_{lower} - d_{upper})$$

$$= 2\Delta y \cdot x_k - 2\Delta x \cdot y_k + c$$

The sign of the decision parameter p_k is the same as that of $d_{lower} - d_{upper}$

If p_k is negative, then we choose the lower pixel, otherwise we choose the upper pixel

Deriving The Bresenham Line Algorithm (cont…)

Remember coordinate changes occur along the x axis in unit steps so we can do everything with integer calculations

At step k+1 the decision parameter is given as:

$$p_{k+1} = 2\Delta y \cdot x_{k+1} - 2\Delta x \cdot y_{k+1} + c$$

Subtracting p_k from this we get:

$$p_{k+1} - p_k = 2\Delta y(x_{k+1} - x_k) - 2\Delta x(y_{k+1} - y_k)$$

Deriving The Bresenham Line Algorithm (cont…)

But, x_{k+1} is the same as x_k+1 so:

$$p_{k+1} = p_k + 2\Delta y - 2\Delta x(y_{k+1} - y_k)$$

where y_{k+1} - y_k is either 0 or 1 depending on the sign of p_k

The first decision parameter p0 is evaluated at (x0, y0) is given as:

$$p_0 = 2\Delta y - \Delta x$$

The Bresenham Line Algorithm

BRESENHAM'S LINE DRAWING ALGORITHM
(for $|m| < 1.0$)

1. Input the two line end-points, storing the left end-point in (x_0, y_0)
2. Plot the point (x_0, y_0)
3. Calculate the constants Δx, Δy, $2\Delta y$, and $(2\Delta y - 2\Delta x)$ and get the first value for the decision parameter as:

$$p_0 = 2\Delta y - \Delta x$$

4. At each x_k along the line, starting at $k = 0$, perform the following test. If $p_k < 0$, the next point to plot is (x_k+1, y_k) and:

$$p_{k+1} = p_k + 2\Delta y$$

The Bresenham Line Algorithm…

Otherwise, the next point to plot is (x_k+1, y_k+1) and:

$$p_{k+1} = p_k + 2\Delta y - 2\Delta x$$

5. Repeat step 4 (Δx – 1) times

Bresenham Example

Let's have a go at this

Let's plot the line from (20, 10) to (30, 18)

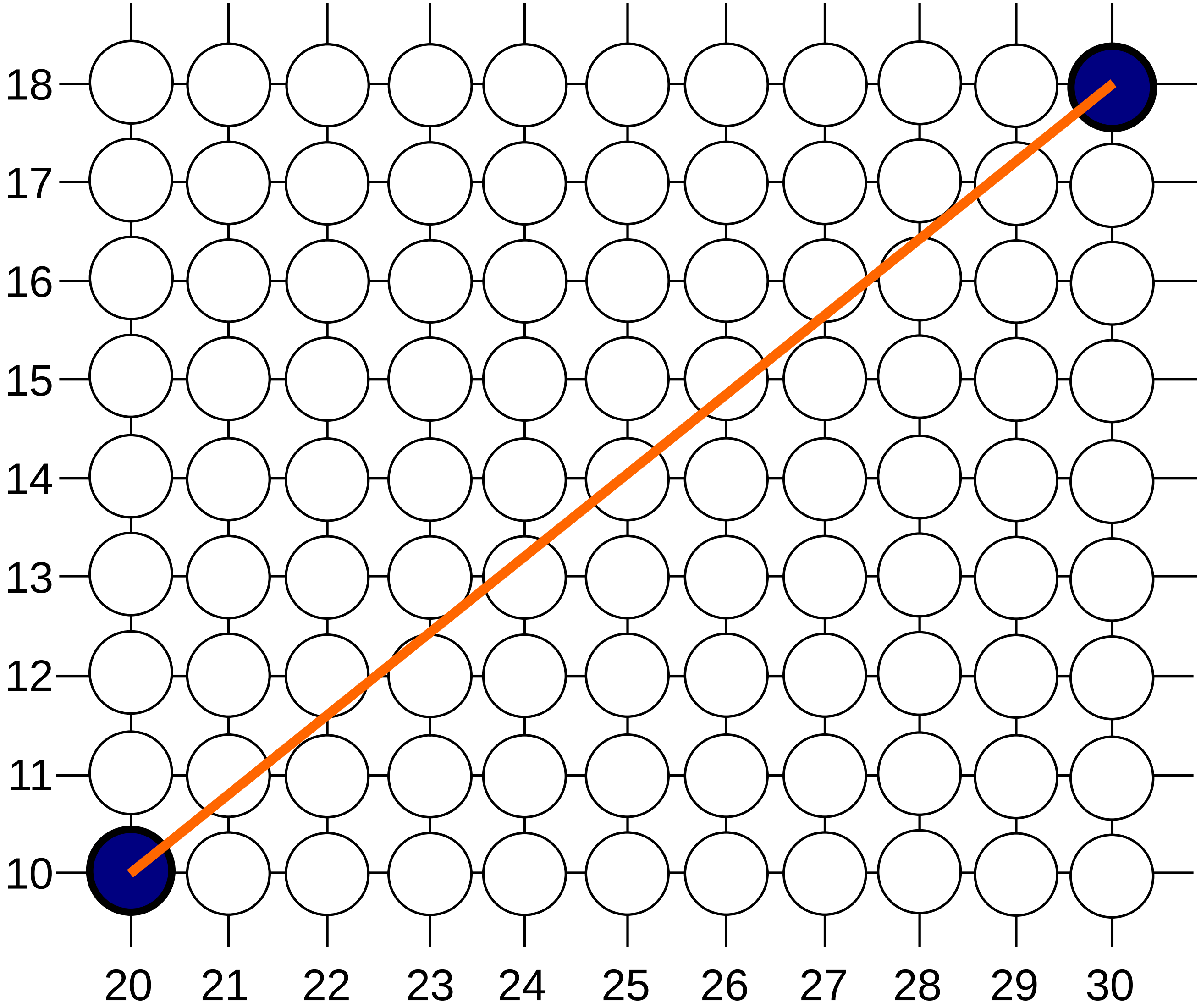
First off calculate all of the constants:

- Δx: 10
- Δy: 8
- $2\Delta y$: 16
- $2\Delta y$ - $2\Delta x$: -4

Calculate the initial decision parameter p_0:

- $p0 = 2\Delta y - \Delta x$ = 6

Bresenham Example...

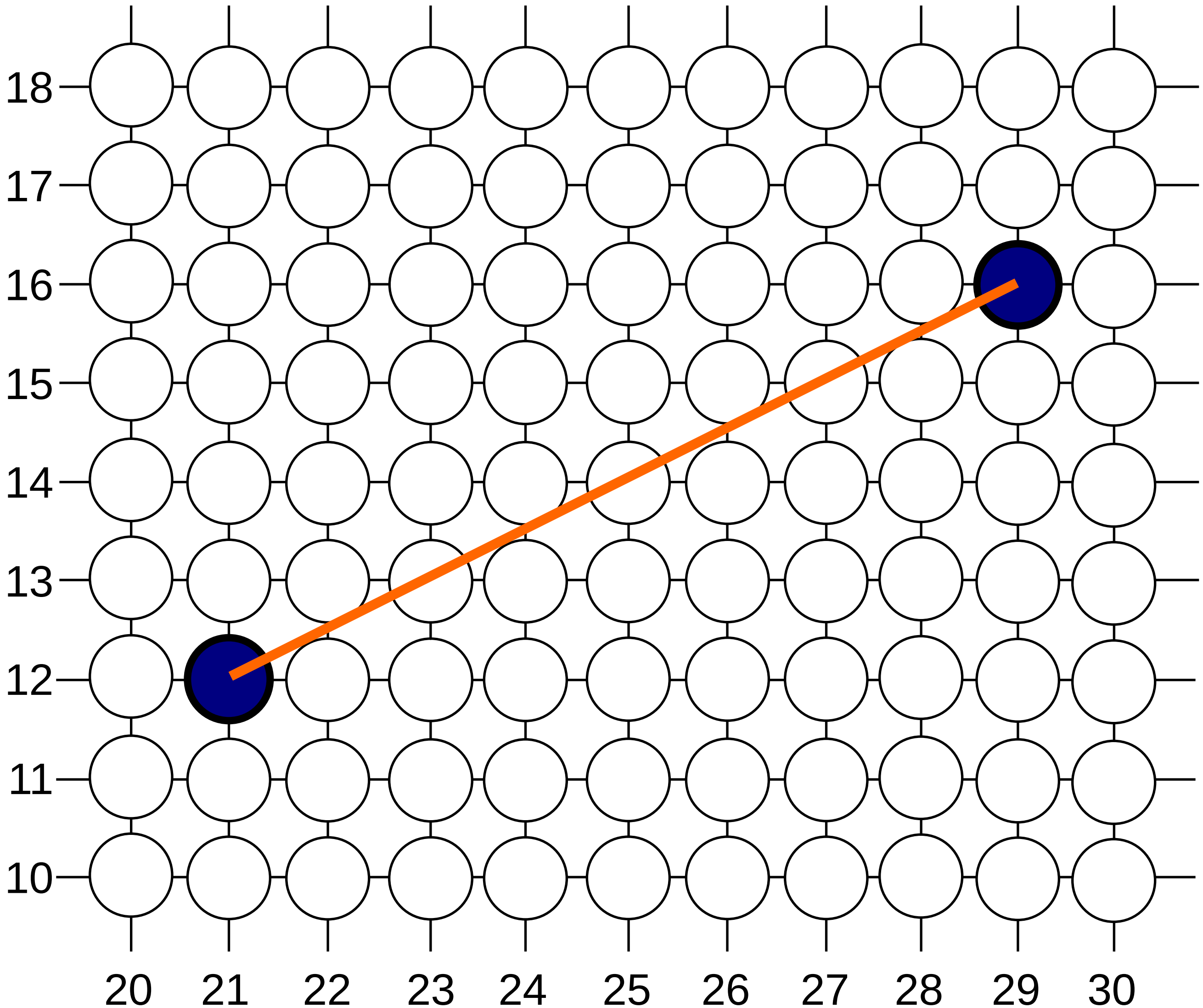


k	p_k	(x_{k+1},y_{k+1})
0		
1		
2		
3		
4		
5		
6		
7		
8		
9		

Bresenham Exercise

Go through the steps of the Bresenham line drawing algorithm for a line going from (21,12) to (29,16)

Bresenham Exercise ...



k	p_k	(x_{k+1},y_{k+1})
0		
1		
2		
3		
4		
5		
6		
7		
8		

Bresenham Line Algorithm Summary

The Bresenham line algorithm has the following advantages:

- A fast incremental algorithm
- Uses only integer calculations

Comparing this to the DDA algorithm, DDA has the following problems:

- Accumulation of round-off errors can make the pixelated line drift away from what was intended
- The rounding operations and floating point arithmetic involved are time consuming

Thanks