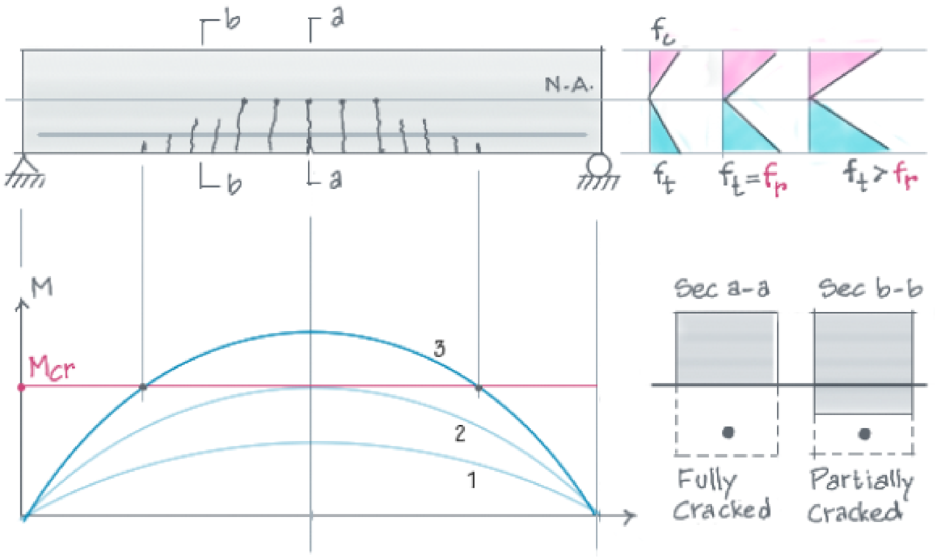
Cracking Moment
(Elastic Analysis of Beam)

Development of Tensile Cracks in RC Beam

Cracking Moment (M_{cr}): The moment at which tensile cracks begin to form, that is, when the tensile stress in the bottom of the beam equals the modulus of rupture (f_r), is referred to as the cracking moment.

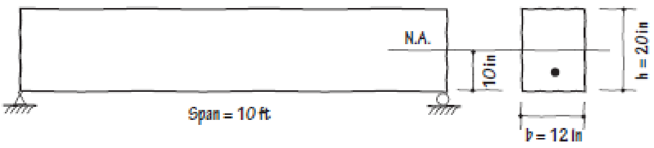
Modulus of rupture (f_r): The bending tensile stress at which the concrete begins to crack.



- At small load, the bending moment diagram (curve 1) is far below the cracking moment (M_{cr}) and the maximum tensile stress (f_t) in the beam is also smaller than modulus of rupture (f_r).
- As load increases, the moment diagram (curve 2) touches M_{cr}, and at that instant, f_t becomes equal to (f_r), thus, crack begins to form at section $a-a$.
- If load is increased further, moment diagram (curve 3) rises beyond M_{cr}, and more cracks are formed on both sides of section $a-a$. Theses cracks propagate vertically until they touch the neutral axis (*NA*).
- If crack propagates fully upto neutral axis, the section is called *fully cracked* section. If crack starts but cannot reach neutral axis, it is called *partially cracked* section. If crack does not start at all, it is called *uncracked* section.

Section Uncracked, Example 1

Ques. Determine if beam has cracked or not. Given that, $f'_c = 4$ ksi, $f_y = 60$ ksi.



Solution.

1. Modulus of rupture, (tensile stress at which concrete cracks)

$$f_r = 7.5\sqrt{f'_c} = 7.5\sqrt{4000} = 474 \text{ psi}$$

Note: While using following formula, $f_r = 7.5\sqrt{f'_c}$, the f'_c must be in psi. That's why $f'_c = 4$ ksi is written as 4000 psi.

2. Moment of inertia of gross section,

$$I_g = \frac{bh^3}{12} = \frac{12\times 20^3}{12} = 8000 \text{ in}^4$$

3. Loading intensity due to self weight,

$$w = \rho A = 150 \text{ lb/ft}^3 \times \frac{12\times 20}{144} \text{ ft}^2 = 250 \text{ lb/ft}$$

Note: The ρ is the density of concrete, which usually equals to 150 lb/ft^3. The $\frac{1}{144}$ is the conversion factor from in^2 to ft^2.

4. Moment at midspan due to self weight

$$M = \frac{wL^2}{8} = \frac{250 \text{ lb/ft}\times 10^2 \text{ ft}^2}{8} = 3125 \text{ lb-ft}$$

5. Stress at the bottom-most point of midspan

$$\sigma = \frac{My}{I_g} = \frac{(3125\times 12 \text{ lb-in})\times 10 \text{ in}}{8000 \text{ in}^4} = 46.87 \text{ psi}$$

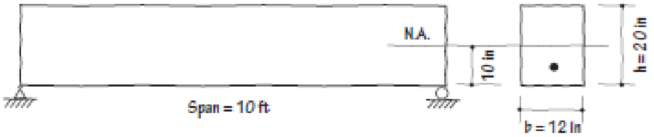
6. Since $\sigma(46.87) < f_r(474)$, no crack has occured yet.

Section Uncracked, Example 2

Ques. If 1.25 k/ft live load is applied on the beam of Example 1, determine whether there is any flexural crack now.

If flexural crack still does not occur, then determine additional load that the beam can carry just before crack occurs.

Given that, $f'_c = 4$ ksi, $f_y = 60$ ksi.



Solution.

Check if concrete has cracked

1. Modulus of rupture, (tensile stress at which concrete crakcs),

$$f_r = 7.5\sqrt{f'_c} = 7.5\sqrt{4000} = 474 \text{ psi}$$

2. Moment of inertia of gross section,

$$I_g = \frac{bh^3}{12} = \frac{12\times20^3}{12} = 8000 \text{ in}^4$$

3. Self weight,

$$w_{\text{self}} = 150\times\frac{12\times20}{144} = 250 \text{ lb/ft} = 0.25 \text{ k/ft}$$

4. Total load per unit length of beam,

$$w_{\text{total}} = w_{\text{self}} + w_{\text{live}} = 0.25 + 1.25 = 1.50 \text{ k/ft}$$

5. Moment at midspan,

$$M = \frac{wL^2}{8} = \frac{1.50\times10^2}{8} = 18.75 \text{ k-ft}$$

6. Stress at the bottom-most point of midspan,

$$\sigma = \frac{My}{I_g} = \frac{(18.75\times12)\times10}{8000} = 0.281 \text{ ksi} = 281 \text{ psi}$$

7. Since $\sigma(281) < f_c(474)$, no crack has occured yet.

Determine additional load

8. Cracking moment,

$$M_{cr} = \frac{f_r I}{y} = \frac{474 \text{ psi}\times8000 \text{ in}^4}{10 \text{ in}} = 379{,}200 \text{ lb-in} = 31.6 \text{ k-ft}$$

Note: This is the same formula, $\sigma = My/I$. If $M = M_{cr}$ then $\sigma = f_r$ and the formula is re-written for M_{cr}. The initial result is divided by 1000 for lb to kip conversion and an additional division by 12 for inch to ft conversion.

9. Total load required for cracking,

$$w = \frac{8M_{cr}}{L^2} = \frac{8\times31.6 \text{ k-ft}}{10^2 \text{ ft}^2} = 2.528 \text{ k/ft}$$

Note: This is the same formula, $M = wL^2/8$, just re-written for w.

10. Total live load required,

$$w_{\text{live}} = w_{\text{total}} - w_{\text{self}} = 2.528 - 0.25 = 2.278 \text{ k/ft}$$

11. Additional live load required,

$$w_{\text{add. live}} = w_{\text{total live}} - w_{\text{existing live}} = 2.278 - 1.25 = 1.028 \text{ k/ft } (Ans.)$$