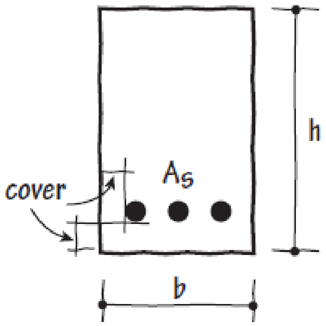
Rectangular Beam Design

Guideline for Beam Design

Designing a beam according to ACI-318 requires to meet a lot of specifications. Many of these are beyond the scope of this text, therefore only the very basic requirements are listed here in simplified form.



Beam Depth

Beam depth (h) is usually around span/12 as a rule of thumb, but ACI-318 suggests to use following table as a guideline for minimum depth. Overall dimensions are selected to whole inches and are incremented by 1 inch to 2 inch to simplify construction process by using uniform formwork.

Simply supported	One end continuous	Both end continuous	Cantilever
$\ell/16$	$\ell/18.5$	$\ell/21$	$\ell/8$

Span length is denoted by ℓ.

Beam Proportions

Unless architectural or other requirements dictate, the depth width ratio is $1\frac{1}{2}$ to 2 for shorter beams (up to 25 ft length). For longer spans, better economy is usually obtained if deep section such as 3 to 4 times of the widths is used

Bar Sizes

For the usual situations, bars of sizes #11 and smaller are practical. It is usually convenient to use bars of one size only in a beam, although occasionally two sizes of bars are be used.

Minimum Reinforcement

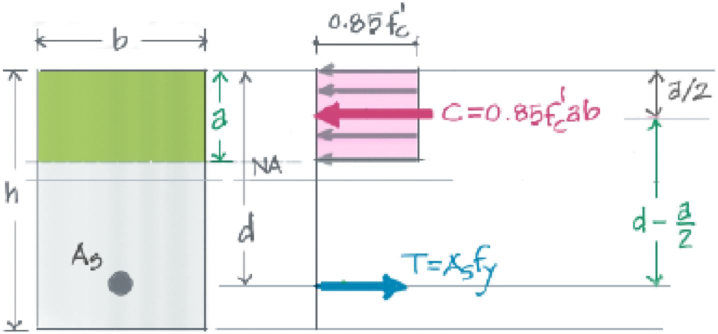
ACI (10.5.1) specifies a certain minimum amount of reinforcing that must be used at every section of flexural members even if required tensile reinforcing is less than minimum.

$$A_{s,min} = \frac{3\sqrt{f'_c}}{f_y} b_w d$$

Cover

To protect reinforcement from fire and corrosion it is located at certain minimum distances from surface of the concrete which is known as cover. The code requires a minimum cover of $1\frac{1}{2}$ in. for beam.

Reinforcement Ratio, Nominal Flexural Resistance Factor



Reinforcement Ratio, (ρ)
Reinforcement ratio is defined as the total amount of tensile area (A_s) over effective concrete area (bd).

$$\rho = \frac{A_s}{bd}; \qquad \boxed{A_s = \rho bd}$$

Nominal Flexural Resistance Factor, (R_n)
Recalling the expression of depth of Whitney's stress block, and substituting $A_s = \rho bd$,

$$a = \frac{A_s f_y}{0.85\,f'_c b} = \frac{(\rho bd)f_y}{0.85\,f'_c b} = \frac{\rho d f_y}{0.85\,f'_c}$$

Recalling expression of design moment capacity, and substituting A_s and a found earlier

$$\phi M_n = M_u = \phi A_s f_y\left(d - \frac{a}{2}\right) = \phi(\rho bd)f_y\left(d - \frac{1}{2}\times\frac{\rho d f_y}{0.85\,f'_c}\right)$$

$$M_u = \phi bd^2 \rho f_y\left(1 - \frac{\rho f_y}{1.7\,f'_c}\right)$$

Rearranging,

$$\frac{M_u}{\phi bd^2} = \rho f_y\left(1 - \frac{\rho f_y}{1.7\,f'_c}\right)$$

The expression on left hand side is defined as nominal flexural resistance factor (R_n). Therefore

$$\boxed{R_n = \frac{M_u}{\phi bd^2}}$$

Substituting R_n,

$$R_n = \rho f_y\left(1 - \frac{\rho f_y}{1.7\,f'_c}\right)$$

Now solving ρ, we find

$$\boxed{\rho = \frac{0.85\,f'_c}{f_y}\left[1 - \sqrt{1 - \frac{2R_n}{0.85\,f'_c}}\right]}$$

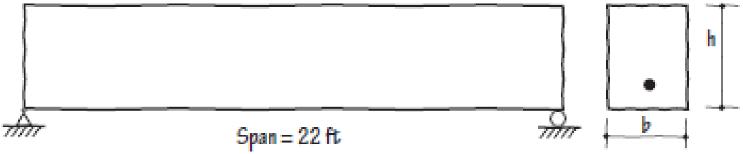
Beam design means that we need to choose the geometric parameters of the beam, such as width (b), height (h), effective depth (d), and provide adequate reinforcing steel (A_s) so that the beam can carry the imposed load upon it safely including its self weight.

Steps in Beam Design

1. Choose beam depth (h), width (b) and effective depth (d)
2. Estimate design load
3. Determine design moment M_u
4. Assume $\phi = 0.90$ and determine R_n
5. Determine ρ by substituting R_n into expression of ρ
6. Find $A_s = \rho bd$
7. Validate the assumption of $\phi = 0.90$
8. Check if beam capacity is greater than design moment, i.e., $\phi M_n \geq M_u$

Example 1, Simply Supported Beam

Ques. Design a rectangular beam of 22 ft length which is required to carry dead load of 1 k/ft and live load of 2 k/ft. Given that, $f'_c = 4$ ksi, $f_y = 60$ ksi.



Estimate Beam Size and Weight

Assume beam height is span/12.

$$h = 22 \text{ ft}/12 = 1.833 \text{ ft} = 22 \text{ in}$$

$$\therefore h = 22 \text{ in.}$$

Estimate effective depth, usually 2.5 ~ 3 inch less than beam height)

$$d = h - 2.5 = 22 - 2.5 = 19.5 \text{ in}$$

Assume beam width, usually 50 ~ 65% of beam height.

$$b = 0.5h = 0.50\times 22 = 11 \text{ in}$$

$$\therefore b = 12 \text{ in.}$$

(Note : We could use b = 11 in, but choosing a nice even number is convenient for construction)

Estimate beam weight per linear feet.

$$w_{self} = \left(\frac{12\times 22}{144} \text{ ft}^2\right)\times 150 \text{ lb/ft}^3 = 275 \text{ lb/ft} = 0.275 \text{ k/ft}$$

Compute Design Load and Moment

$$w_u = 1.2D + 1.6L$$
$$= 1.2\times(1 + 0.275) + 1.6\times 2$$
$$= 4.73 \text{ k/ft}$$

$$M_u = (w_u L^2)/8 = (4.73\times 22^2)/8$$
$$= 286.2 \text{ k-ft}$$

Determine Reinforcement Area (Assuming $\phi = 0.90$)

$$R_n = \frac{M_u}{\phi b d^2} = \frac{286.2\times 12 \text{ k-in}}{0.9\times 12 \text{ in}\times(19.5 \text{ in})^2}$$
$$= 0.836 \text{ ksi}$$

$$\rho = \frac{0.85 f'_c}{f_y}\left[1 - \sqrt{1 - \frac{2R_n}{0.85 f'_c}}\right]$$
$$= \frac{0.85\times 4}{60}\left[1 - \sqrt{1 - \frac{2\times 0.836}{0.85\times 4}}\right]$$
$$= 0.0163$$

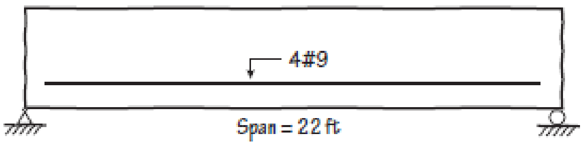
$$A_s = \rho b d = 0.0163\times 12 \text{ in}\times 19.5 \text{ in}$$
$$= 3.81 \text{ in}^2$$

We need to provide reinforcement so that it has at least an area of 3.81 in^2. We know that each No.9 bar has an area of 1.00 in^2. If we choose 4 of them, total area would be 4.00 in^2 which is greater than required 3.81 in^2.

Use 4#9 bars ($A_s = 4\times 1.00 = 4.00$ in^2)

Example 1, Simply Supported Beam

Note : Other bar choice is also possible. For example, each No. 10 bar has an area of 1.27 in^2, therefore, three of them would be, $3\times1.27 = 3.81\ in^2$, which is exactly what we need. Thus, using 3# 10 bar is also an option. Choosing the appropriate bar size falls into the category of structural detailing which is far beyond the scope of this text. Author feels that choosing No. 9 bar serves the purpose of this example, moreover it has an area of exactly 1.00 in^2 which makes the calculation easier.



Now validate the assumption of $\phi = 0.90$.

Check Strain

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{4.00\times60}{0.85\times4\times12} = 5.88 \text{ in}$$

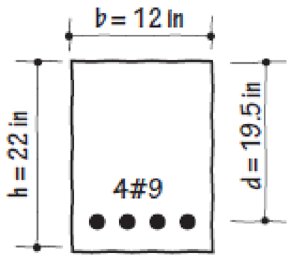
$$c = \frac{a}{\beta_1} = \frac{5.88}{0.85} = 6.92 \text{ in}$$

$$(\beta_1 = 0.85, \text{ since } f'_c \le 4000 \text{ psi})$$

$$\epsilon_t = \left(\frac{d-c}{c}\right)\epsilon_u$$

$$= \left(\frac{19.5-6.92}{6.92}\right)0.003 = 0.0054$$

Since $\epsilon_t > 0.005$, so $\phi = 0.90$ was ok.



Designed Section

Check Capacity

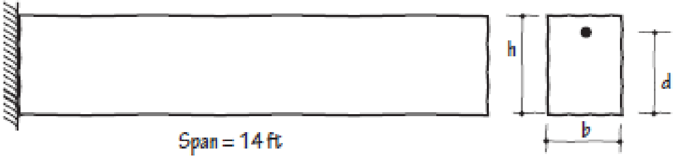
$$\phi M_n = \phi A_s F_y\left(d - \frac{a}{2}\right)$$

$$= 0.9\times4\times60\times\left(19.5 - \frac{5.88}{2}\right)$$

$$= 3577 \text{ k-in} = 298.1 \text{ k-ft} > M_u(286.2) \quad \text{O.K.}$$

Example 2, Cantilever Beam

Ques. Design a rectangular cantilever beam of 14 ft length which is required to carry dead load of 0.5 k/ft and live load of 1.0 k/ft. Given that, $f'_c = 4$ ksi, $f_y = 60$ ksi.



Since this is a cantilever beam, tensile reinforcement should be at top of neutral axis for the entire beam. The effective depth (d) should be measured from bottom.

Estimate Beam Size and Weight

Assume beam height, span/8 for cantilever beam.

$$h = 14/8 = 1.75 \text{ ft} = 21 \text{ in}$$

$$\therefore h = 22 \text{ in.}$$

(Note: It's nice to choose even number for beam sizes.)

Estimate effective depth.

$$d = h - 2.5 = 22 - 2.5 = 19.5 \text{ in}$$

Assume beam width, usually 50 ~ 60% of beam height.

$$b = 0.5h = 0.5\times 22 = 11 \text{ in}$$

$$\therefore b = 12 \text{ in.}$$

Estimate beam weight per linear feet.

$$w_{self} = \frac{12\times 22}{144}\times 150 = 275 \text{ lb/ft} = 0.275 \text{ k/ft}$$

Compute Design Load and Moment

$$w_u = 1.2D + 1.6L$$
$$= 1.2\times(0.5 + 0.275) + 1.6\times 1.0$$
$$= 2.53 \text{ k/ft}$$

$$M_u = (w_u L^2)/2 = (2.53\times 14^2)/2$$
$$= 247.9 \text{ k-ft}$$

(Note: Unlike simply supported beam, cantilever beam has maximum moment at support which equals $wL^2/2$. The maximum moment in simply supported beam (like the previous example), occurs at midspan which equals $wL^2/8$.

Determine Reinforcement (Assuming, $\phi = 0.90$)

$$R_n = \frac{M_u}{\phi b d^2} = \frac{247.9\times 12}{0.9\times 12\times 19.5^2}$$
$$= 0.724 \text{ ksi}$$

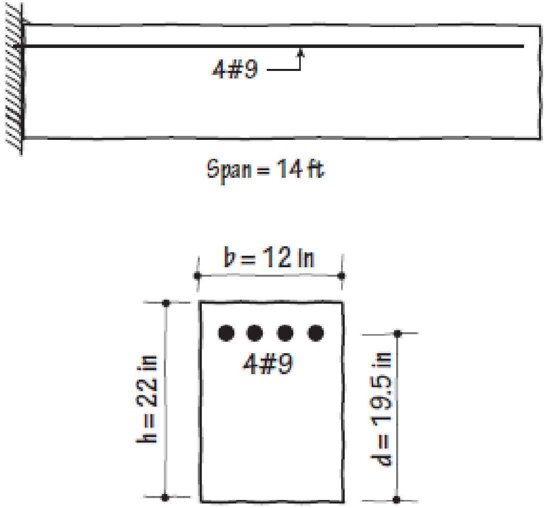
$$\rho = \frac{0.85 f'_c}{f_y}\left[1 - \sqrt{1 - \frac{2R_n}{0.85 f'_c}}\right]$$
$$= \frac{0.85\times 4}{60}\left[1 - \sqrt{1 - \frac{2\times 0.724}{0.85\times 4}}\right]$$
$$= 0.0137$$

$$A_s = \rho b d = 0.0137\times 12\times 19.5$$
$$= 3.21 \text{ in}^2$$

Use 4#9 bars ($A_s = 4\times 1.00 = 4.00$ in^2)

Example 2, Cantilever Beam

Now validate the assumption of $\phi = 0.90$.



Designed Section

Check Strain

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{4.00 \times 60}{0.85 \times 4 \times 12} = 5.88 \text{ in}$$

$$c = \frac{a}{\beta_1} = \frac{5.88}{0.85} = 6.92 \text{ in}$$

$$(\beta_1 = 0.85, \text{ since } f'_c \leq 4000 \text{ psi})$$

$$\epsilon_t = \left(\frac{d-c}{c}\right)\epsilon_u$$

$$= \left(\frac{19.5 - 6.92}{6.92}\right)0.003 = 0.00545$$

Since $\epsilon_t > 0.005$, so $\phi = 0.90$ was ok.

Check Capacity

$$\phi M_n = \phi A_s F_y \left(d - \frac{a}{2}\right)$$

$$= 0.90 \times 4 \times 60 \times \left(19.5 - \frac{5.88}{2}\right)$$

$$= 3576 \text{ k-in} = 298.1 \text{ k-ft} > M_u(247.9) \quad \text{O.K.}$$