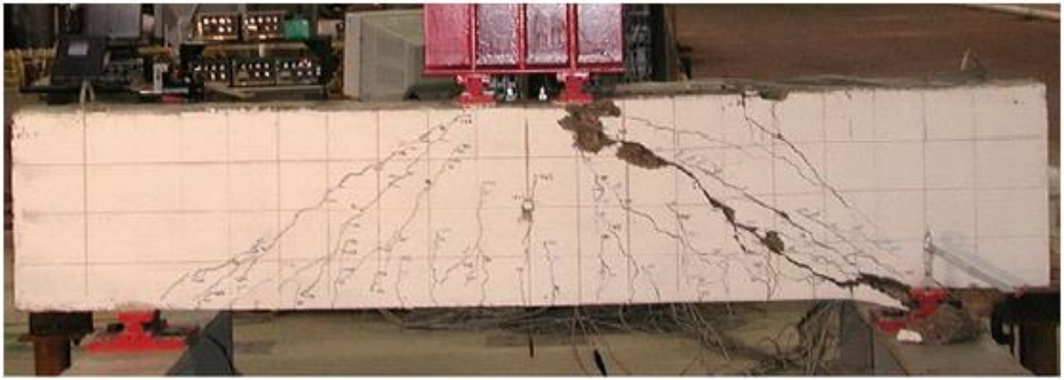
Shear Design

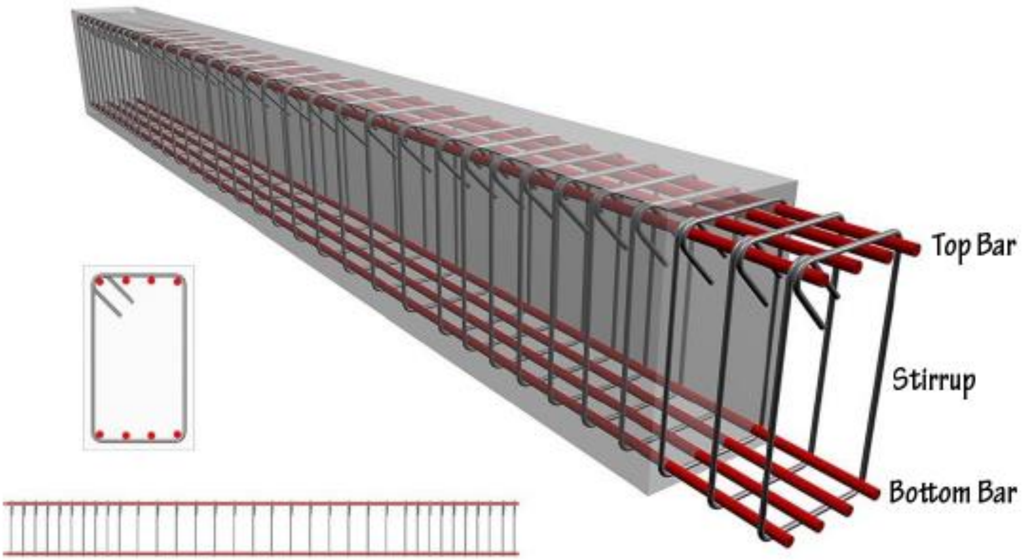
Flexural and Shear Cracks in Beam

Vertical cracks are formed due to flexure, which are known as *flexural crack*



Inclined cracks are formed due to shear, which are known as *shear crack*

Typical Reinforcement Layout of Beam



- To resist *tensile forces* that occur at both top part and bottom part of a section; *top bars* and *bottom bars* are provided respectively. These bars are also known as *longitudinal reinforcement*.
- *Shear forces* and *torsional forces* are resisted by *stirrups* which are also called as *transverse reinforcement*.
- Unlike top bars and bottom bars, size of stirrups are *selected* rather than *determined*. Usually No. 3 and No. 4 bars are selected for stirrups.
- Based on the selected bar for stirrups, the spacing is then *determined*.

Shear Strength of Concrete and Shear Force Resisted by Steel

Shear of Strength of Concrete (V_c)

- Concrete is very weak in resisting tension, but it's shear strength is significantly higher comparing to its tensile strength.
- ACI-318 suggests to use the following formula to determine concrete's shear strength,

$$V_c = 2\sqrt{f'_c}bd$$

- It's a common mistake to include *gross area* of concrete (bh) instead of an *effective area* of concrete (bd) in the above formula.
- The unit of f'_c in the above formula must be in psi. For example, if $f'_c = 4$ ksi is given, then it should be changed to $f'_c = 4000$ psi while plugging it into the formula.

Shear Force Resisted by Steel (V_s)

- Total shear strength of a concrete beam is denoted by V_n.
- Concrete can provide upto V_c shear force.
- The remaining shear force is provided by steel, which is denoted by V_s. Thus,

$$V_n = V_c + V_s$$

- Multiplying by ϕ on both side,

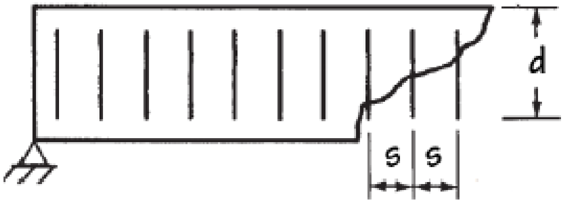
$$\phi V_n = \phi V_c + \phi V_s$$

$$V_u = \phi V_c + \phi V_s$$

$$V_s = \frac{V_u - \phi V_c}{\phi}$$

- Here, V_u is the design shear force and $\phi = 0.75$ for shear.
- According to ACI-318, V_u should be calculated at d distance from support, but for the sake of simplicity in this text, V_u will be determined at support.
- Therefore, V_u is nothing but the support reaction. For simply supported beam, $V_u = w_u L/2$ and for cantilever beam, $V_u = w_u L$.

Determination of Stirrup Spacing



Symb.	Description
V_c	Shear strength of concrete
V_s	Shear force resisted by steel
V_u	Design strength
A_v	Contributing area of a single stirrup to resist shear force
f_y	Yield strength of steel of stirrup
d	Effective depth of beam
s	Spacing of stirrups

1. Find shear capacity of concrete, $\phi = 0.75$ for shear

$$\phi V_c = \phi 2 \sqrt{f_c'} bd$$

2. Find amount of shear that must be carried by steel

$$V_s = \frac{V_u - \phi V_c}{\phi}$$

3. Determine spacing of stirrup

$$s = \frac{A_v f_y d}{V_s}$$

4. Spacing cannot be greater than $d/2$ or 24 inch

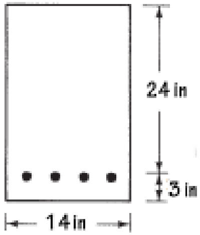
Note: According to ACI-318 11.4.5.3, there is another $d/4$ or 12 inch version of the above formula which is skipped for the sake of simplicity in this text.

The contributing area of stirrup (A_v) is equal to two times of the cross sectional area of the stirrup. Because, if an imaginary section (horizontal, vertical or even inclined) is passed through a stirrup, it will intersect at two cross sections of the stirrup.

For example, a No. 3 bar has cross sectional area of 0.11 in^2. If No. 3 bar is used for stirrup, then $A_v = 2 \times 0.11 = 0.22$ in^2.

Example 1, 2

Ques. If $V_u = 40$ kip, then determine spacing of #3 stirrup for the section. Given that, $f'_c = 3$ ksi and $f_y = 60$ ksi.



Solution.

1. Find shear capacity of concrete

$$\phi V_c = \phi 2\sqrt{f'_c}bd$$

$$= 0.75 \times 2\sqrt{3000} \times 14 \times 24 = 27,605 \text{ lb} = 27.6 \text{ kip}$$

2. Find amount of shear that must be carried by steel

$$V_s = \frac{V_u - \phi V_c}{\phi} = \frac{40 - 27.6}{0.75} = 16.53 \text{ kip}$$

3. Determine spacing of stirrup

$$s = \frac{A_v f_y d}{V_s} = \frac{(2\times0.11)\times60\times24}{16.53} = 19.17 \text{ in}$$

4. Check maximum spacing

$$s_{max} = \begin{cases} d/2 = 24/2 = 12 \text{ in (governs)} \\ 24 \text{ in} \end{cases}$$

Spacing should be the minimum value among 19.17 in, 12 in and 24 in.

Ans. $s = 12$ in

Ques. If $V_u = 80$ kip for the same section, then determine spacing of stirrup.

Solution.

1. Find shear capacity of concrete

$$\phi V_c = \phi 2\sqrt{f'_c}bd$$

$$= 0.75 \times 2\sqrt{3000} \times 14 \times 24 = 27,605 \text{ lb} = 27.6 \text{ kip}$$

2. Find amount of shear that must be carried by steel

$$V_s = \frac{V_u - \phi V_c}{\phi} = \frac{80 - 27.6}{0.75} = 69.87 \text{ kip}$$

3. Determine spacing of stirrup

$$s = \frac{A_v f_y d}{V_s} = \frac{(2\times0.11)\times60\times24}{69.87} = 4.53 \text{ in (governs)}$$

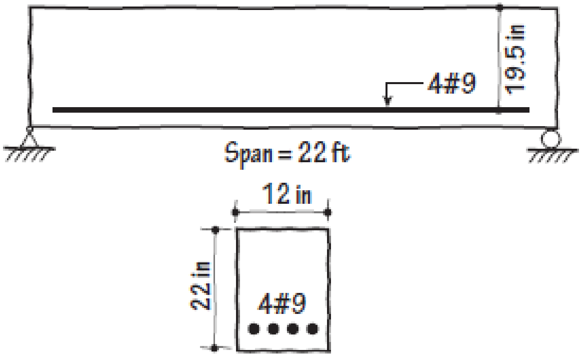
4. Check maximum spacing

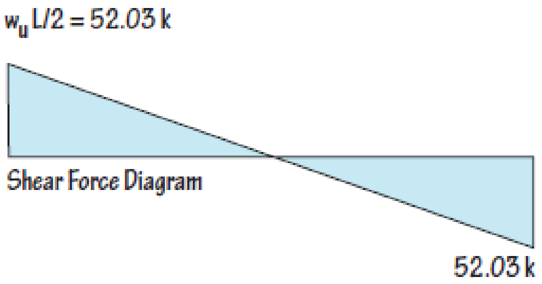
$$s_{max} = \begin{cases} d/2 = 24/2 = 12 \text{ in} \\ 24 \text{ in} \end{cases}$$

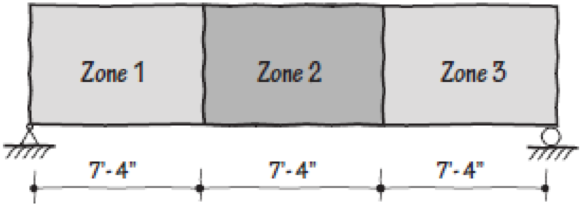
The spacing 4.53 inch is the minimum value among 4.53 in, 12 in and 24 in. Therefore, we could select, $s = 4.53$ in. However, measuring such dimension is error prone and time consuming in construction. That's why nice rounded numbers are preferable. For stirrups, it is practiced that spacing should be *lower rounded to nearest 0.5 inch*.

Ans. $s = 4.5$ in

Example 3







Ques. Design the beam for shear using #3 stirrup. This beam was previously designed for flexure (*Ex. 1 of Flexural Design*). Design load, $w_u = 4.73$ k/ft was determined earlier and f'_c was 4 ksi.

Solution.

It is commonly practiced for concrete beam to divide it into three equal zones and then determine stirrup spacing for each zone.

- The stirrup spacing in Zone 1 will be determined using shear at the support $V_u = w_u L/2$ (i.e., support reaction).
- For the Zone 2, it is customary that the stirrup spacing should be *double* of the spacing determined for Zone 1.
- For Zone 3, the spacing would be same as Zone 1, since the shear diagram is symmetric in this example.

Stirrup Spacing in Zone 1

1. Find design shear which occurs at support

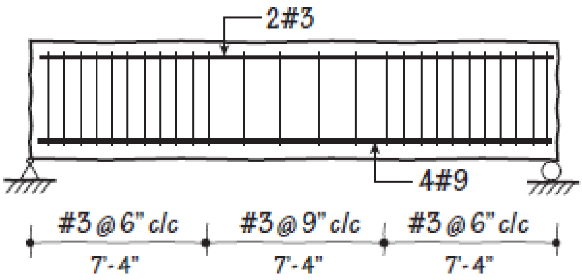
$$V_u = \frac{w_u L}{2} = \frac{4.73 \times 22}{2} = 52.03 \text{ kip}$$

2. Find shear capacity of concrete

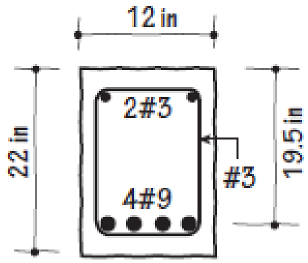
$$\phi V_c = \phi 2 \sqrt{f'_c} bd$$

$$= 0.75 \times 2 \sqrt{4000} \times 12 \times 19.5 = 22,199 \text{ lb} = 22.2 \text{ kip}$$

Example 3



Longitudinal Section



Cross Section

3. Find amount of shear that must be carried by steel

$$V_s = \frac{V_u - \phi V_c}{\phi} = \frac{52.03 - 22.2}{0.75} = 39.8 \text{ kip}$$

4. Determine spacing of stirrup

$$s = \frac{A_v f_y d}{V_s} = \frac{(2\times0.11)\times60\times19.5}{39.8} = 6.46 \text{ in}$$

5. Check maximum spacing

$$s_{max} = \begin{cases} d/2 = 19.5/2 = 9.75 \text{ in} \\ 24 \text{ in} \end{cases}$$

6. Provide #3 @ 6 inch c/c at Zone 1.

Stirrup Spacing in Zone 2 and Zone 3

- For Zone 2, spacing could be the double of Zone 1, i.e., $2\times6 = 12$ in. But that crosses the limit of maximum spacing, $d/2 = 9.75$ in. Therefore, we cannot provide 12 inch. Thus selecting 9 inch. (Another option would be 9.50 inch.)
- In Zone 3, the spacing would be same as Zone 1, i.e., 6 inch.

Hanger Bars

Other than stirrups, we need to provide 2 hanger bars. These bars are not intended to carry any tension or compression but are provided to attach the stirrups with it. Since strength is not a concern, we could just select the thinnest available bar, i.e., #3 bar.