Analysis of Doubly Reinforced Beam

Design Capacity of Doubly Reinforced Beam

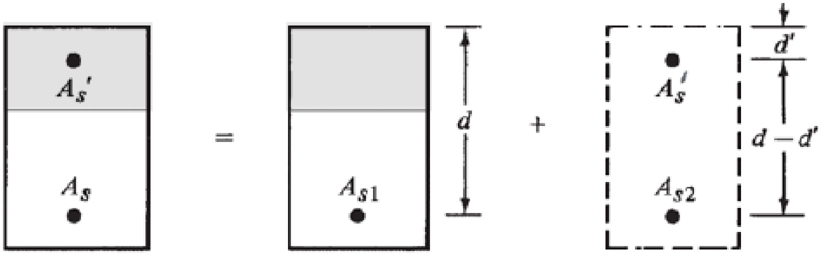


Fig. (a) Fig. (b) Fig. (c)

- **Fig. (a):** Beams with both tensile and compressive steel are referred to as *doubly reinforced beams*. The steel that is used on the compression sides of beams is called compression steel (A'_s) and the steel on the tension side is called tensile steel (A_s).
- The nominal resisting moment of the beam is assumed to consist of two parts, shown in Fig.(b) and Fig.(c).
- **Fig. (b):** First part is the moment (M_{n1}) resisted by compression concrete (shaded gray) and the balancing tensile reinforcing (A_{s1}).

$$M_{n1} = A_{s1} f_y \left(d - \frac{a}{2}\right)$$

- **Fig. (c):** Second part is the moment (M_{n2}) resisted by the compression steel (A'_s) and the balancing amount of the additional tensile steel (A_{s2}).

$$M_{n2} = A_{s2} f'_s (d - d')$$

- The total design moment capacity is found by adding the two nominal capacities and multiplying by ϕ.

$$\phi M_n = \phi(M_{n1} + M_{n2})$$

$$\phi M_n = \phi\left[A_{s1} f_y \left(d - \frac{a}{2}\right) + A_{s2} f'_s (d - d')\right]$$

Sym.	Description	Remarks
A_s	Total tensile steel area, $A_s = A_{s1} + A_{s2}$	Known
A'_s	Compression steel area	Known
A_{s1}	Tensile steel area that balances concrete compression	Unknown
A_{s2}	Tensile steel area that balances A'_s	Unknown
f_y	Stress in tensile steel	Known
f'_s	Stress in compressive steel	Unknown

Determination of A_{s1} and A_{s2} (If Compression Steel Yields) Doubly Reinforced Beam

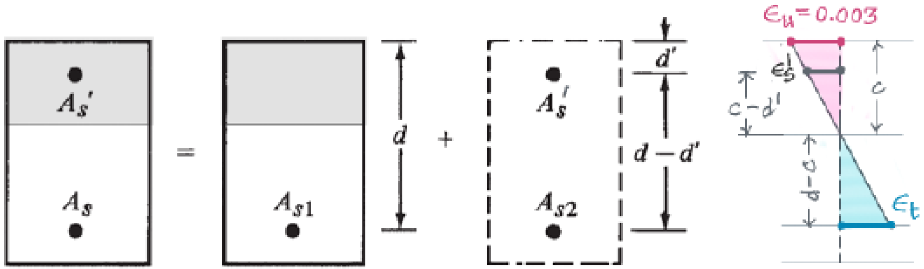


Fig. (a) Fig. (b) Fig. (c) Fig. (d)

- **Fig. (c):** By writing the force equilibrium equation,

$$C = T; \qquad \boxed{A_{s2} f_y = A_s' f_s'}$$

- The tensile steel always yields (f_y), but the stress in compression steel (f_s') may reach yield strength or not.
- **Fig. (d):** To determine f_s', we must know the strain in compression steel (ϵ_s'). From the two similar triangles in the compressive zone (magenta), we write,

$$\frac{\epsilon_s'}{c - d'} = \frac{\epsilon_u}{c}; \qquad \boxed{\epsilon_s' = \left(\frac{c - d'}{c}\right)\epsilon_u}$$

- If, $\epsilon_s' \geq \epsilon_y$, then compression steel has yielded, if $\epsilon_s' < \epsilon_y$, then compression steel has not yielded. Here, ϵ_y is the yield strain of steel.
- If, $\epsilon_s' \geq \epsilon_y$, then f_s' becomes equal to f_y. Therefore,

$$A_{s2} f_y = A_s' f_s'; \qquad A_{s2} f_y = A_s' f_y; \qquad \boxed{A_{s2} = A_s'}$$

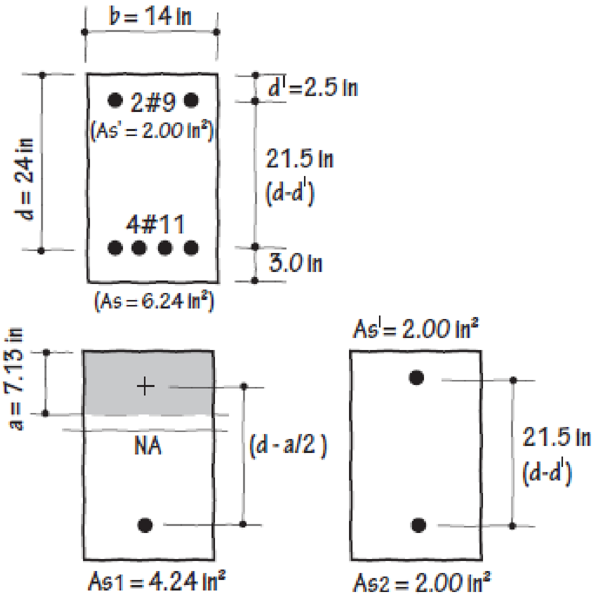
- The A_{s1} is now easily found using following equation,

$$A_s = A_{s1} + A_{s2}; \qquad \boxed{A_{s1} = A_s - A_{s2}}$$

Note: The case of compression steel not yielded is beyond of the scope of this text.

Example 1

Ques. Determine design moment capacity of the doubly reinforced beam. Given that, *compressive steel has yielded and* $\epsilon_t > 0.005$. Take, $f'_c = 3$ ksi, $f_y = 60$ ksi.



Solution.

From the given beam,

$$A_s = 4\times1.56 = 6.24 \text{ in}^2$$

$$A'_s = 2\times1.00 = 2.00 \text{ in}^2$$

Find Depth of Whitney's Stress Block

It is given that compression steel has yielded, therefore, $f'_s = f_y = 60$ ksi.

$$T = C; \qquad T_{steel} = C_{conc} + C_{comp.\ steel}$$

$$A_s f_y = 0.85\, f'_c ab + A'_s f'_s$$

$$6.24\times60 = 0.85\times3\times a\times14 + 2.00\times60$$

$$a = 7.13 \text{ in}$$

Determine Steel Area A_{s1} and A_{s2}

The formula $A'_s = A_{s2}$ applies if compression steel yields.

$$A_{s2} = A'_s = 2.00 \text{ in}^2$$

$$A_{s1} = A_s - A_{s2} = 6.24 - 2.00 = 4.24 \text{ in}^2$$

Determine Capacity

It is given that $\epsilon_t > 0.005$, therefore, $\phi = 0.90$.

$$\phi M_n = \phi\left[A_{s1} f_y \left(d - \frac{a}{2}\right) + A'_s f'_s (d - d')\right]$$

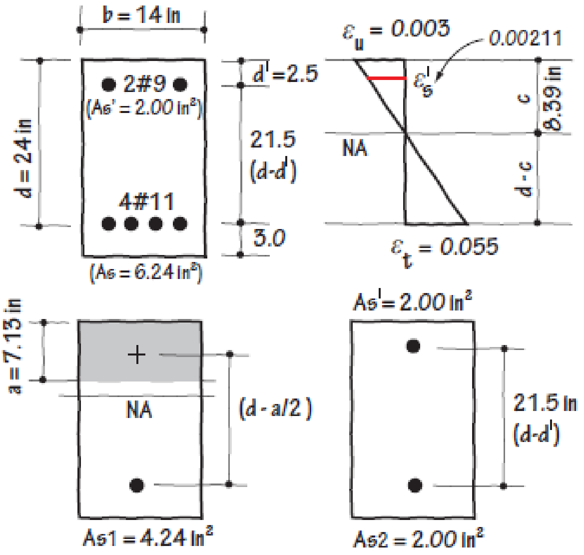
$$= 0.90\left[4.24\times60\left(24 - \frac{7.13}{2}\right) + 2\times60\times(24 - 2.5)\right]$$

$$= 7000 \text{ k-in} = 583.4 \text{ k-ft } (Ans.)$$

Example 2

Doubly Reinforced Beam

Ques. Determine design moment capacity of the doubly reinforced beam. Given that, $f'_c = 3$ ksi, $f_y = 60$ ksi.



Solution.

It's the same beam of the previous problem, Ex. 1. But it is not given that the compression steel yields and the validity of $\epsilon_t > 0.005$ is also unknown. We must verify these ourselves.

It will be assumed that compression steel has yielded, $f'_s = f_y = 60$ ksi, and the assumption will be checked later.

Find Depth of Whitney's Stress Block

$$A_s f_y = 0.85\, f'_c a b + A'_s f'_s$$

$$(4\times 1.56)\times 60 = 0.85\times 3\times a\times 14 + 2.00\times 60; \qquad a = 7.13 \text{ in}$$

Find Depth of Neutral Axis

$$c = \frac{a}{\beta} = \frac{7.13}{0.85} = 8.39 \text{ in}$$

Check Compression Steel Strain

$$\epsilon'_s = \left(\frac{c-d'}{c}\right)\epsilon_u = \left(\frac{8.39-2.5}{8.39}\right)0.003 = 0.00211$$

$$\epsilon_y = \frac{f_y}{E} = \frac{60 \text{ ksi}}{29000 \text{ ksi}} = 0.00206$$

Since, $\epsilon'_s > \epsilon_y$, compression steel has yielded. Therefore, the assumption, $f'_s = f_y = 60$ ksi, was correct.

Determine Steel Area A_{s1} and A_{s2}

$$A_{s2} = A'_s = 2.00 \text{ in}^2$$

$$A_{s1} = A_s - A_{s2} = 6.24 - 2.00 = 4.24 \text{ in}^2$$

Check Tensile Steel Strain

$$\epsilon_t = \left(\frac{d-c}{c}\right)\epsilon_u = \left(\frac{24-8.39}{8.39}\right)0.003 = 0.0055$$

Since $\epsilon_t > 0.005$, therefore, $\phi = 0.90$.

Determine Capacity

$$\phi M_n = \phi\left[A_{s1} f_y \left(d - \frac{a}{2}\right) + A'_s f_y (d - d')\right]$$

$$= 0.90\left[4.24\times 60\left(24 - \frac{7.13}{2}\right) + 2\times 60\times(24-2.5)\right]$$

$$= 7000 \text{ k-in} = 583.4 \text{ k-ft } (Ans.)$$